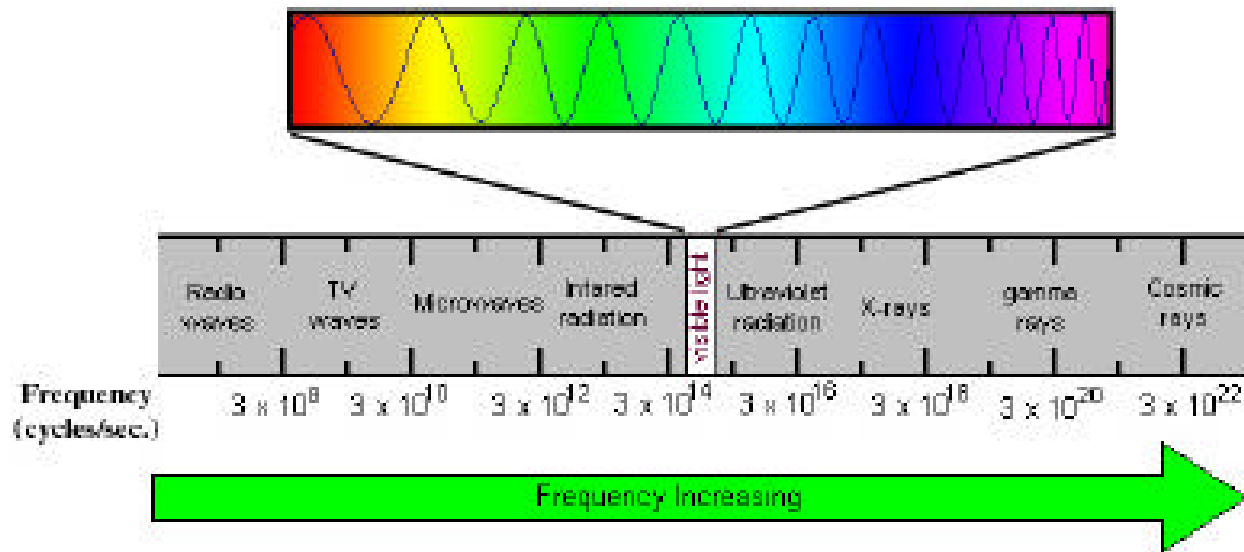


Digital television

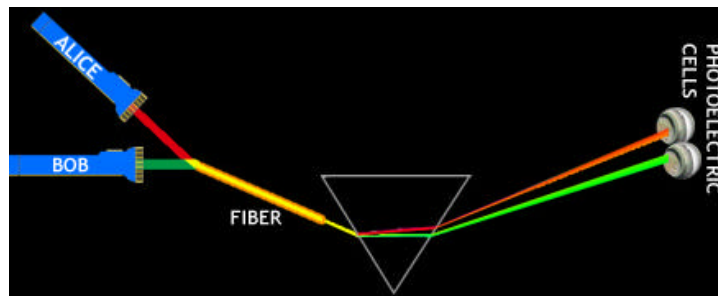
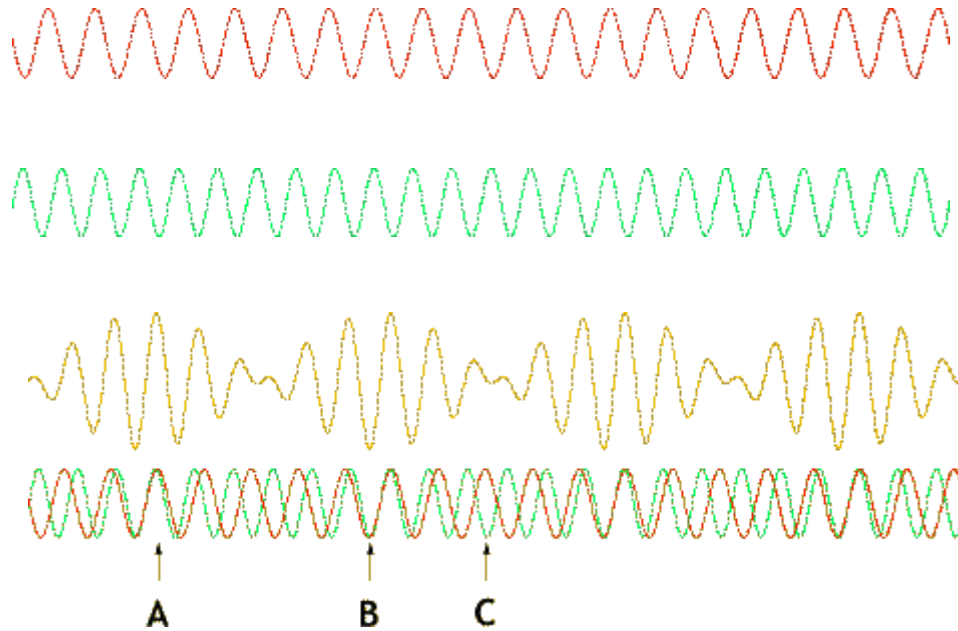
Modulation techniques

- Electromagnetic waves
- Analog modulation
 - Amplitude modulation
 - Angle modulation
 - Frequency modulation
 - Phase modulation
- Digital modulation
 - On-Off keying
 - Amplitude shift keying
 - Phase shift keying
 - Quadrature amplitude modulation

Electromagnetic waves



Electromagnetic waves



Modulation

"modulate" is "To adjust or adapt to a certain proportion."

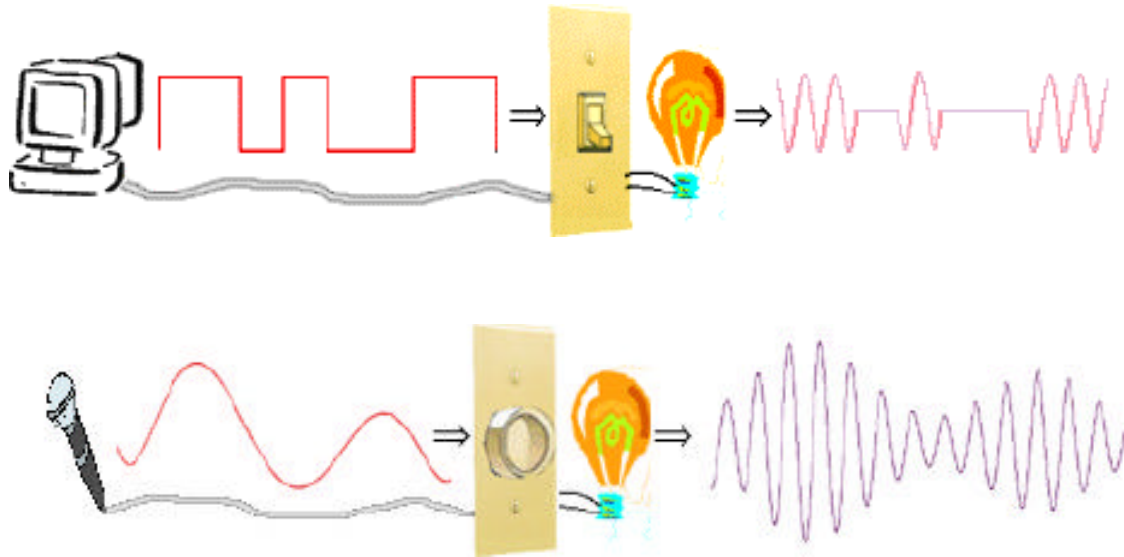
"Linear" modulation

$$x_c(t) = A_c (1 + \mu x(t)) \cos \omega_c t$$

Exponential modulation

$$\begin{aligned} x_c(t) &= A_c \cos(\omega_c t + \phi(t)) \\ &= A_c \cos \Theta_c(t) = A_c \operatorname{Re}[e^{j\Theta_c(t)}] \end{aligned}$$

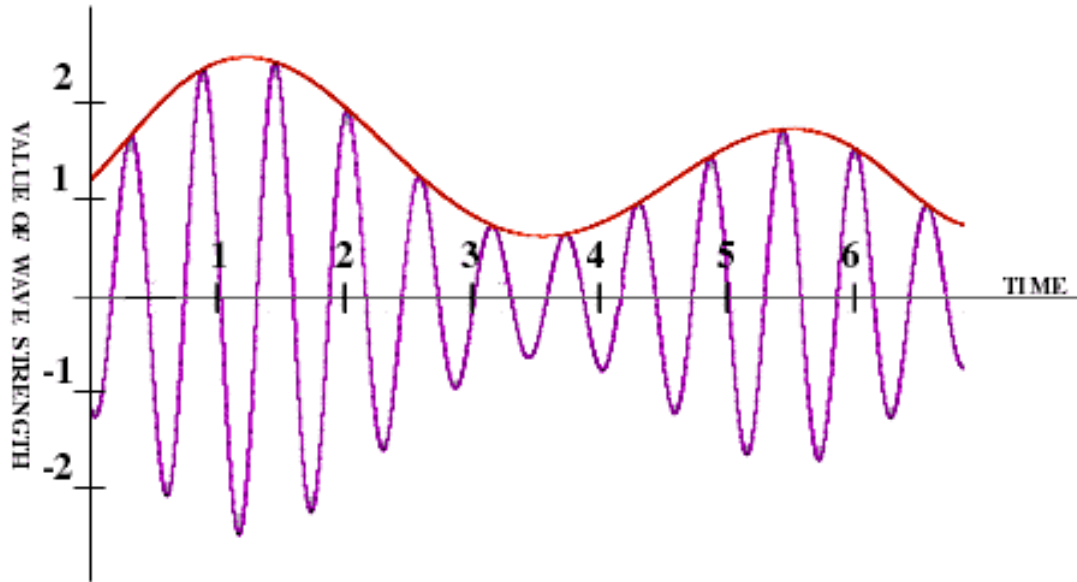
Amplitude modulation



E.g..

- AM radio stations
- Analog television

Amplitude modulation



$$x_c(t) = A_c m(t) \cos 2\pi f_c t \quad (3.1)$$

$$X_c(f) = \frac{A_c}{2} [M(f + f_c) + M(f - f_c)] \quad (3.2)$$

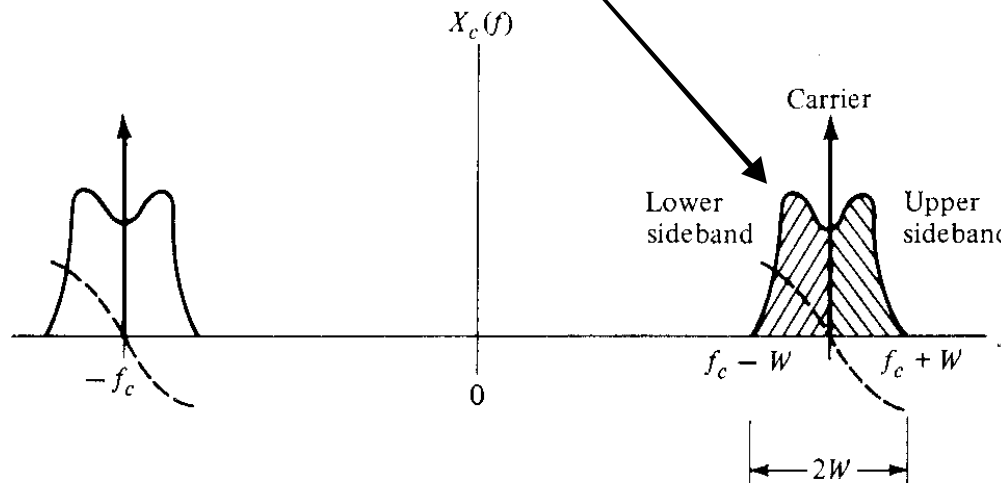
AM: waveforms and bandwidth

- AM in frequency domain:

$$\begin{aligned}
 x_c(t) &= A_c [1 + m x_m(t)] \cos(w_c t) \\
 &= \underbrace{A_c \cos(w_c t)}_{\text{Carrier}} + \underbrace{m x_m(t) \cos(w_c t)}_{\text{Information carrying part}}
 \end{aligned}$$

$$X_c(f) = \underbrace{A_c d(f - f_c) / 2}_{\text{Carrier}} + \underbrace{m A_c X_m(f - f_c) / 2}_{\text{Information carrying part}} \quad f > 0 \text{ (for brief notations)}$$

- AM bandwidth is twice the message bandwidth W :



Phase modulation (PM)

Carrier Wave (CW) signal: $x_c(t) = A_c \cos(\underbrace{w_c t + f(t)}_{q_c(t)})$

In exponential modulation the modulation is “in the exponent” or “in the angle”

$$x_c(t) = A_c \cos(q_c(t)) = A_c \operatorname{Re}[\exp(jq_c(t))]$$

Note that in exponential modulation superposition does not apply:

$$x_c(t) = A \cos\{w_c t + k_f [a_1(t) + a_2(t)]\}$$

$$\neq A \cos w_c t + A \cos k_f [a_1(t) + a_2(t)]$$

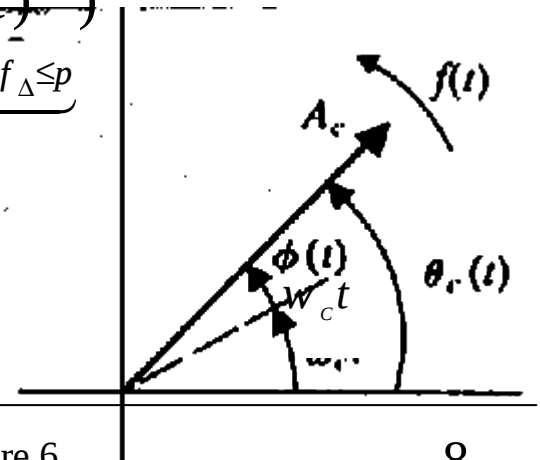
In **phase modulation** (PM) carrier phase is linearly proportional to the modulation amplitude:

$$x_{PM}(t) = A_c \cos(w_c t + \underbrace{f(t)}_{f_{\Delta} x(t), f_{\Delta} \leq p})$$

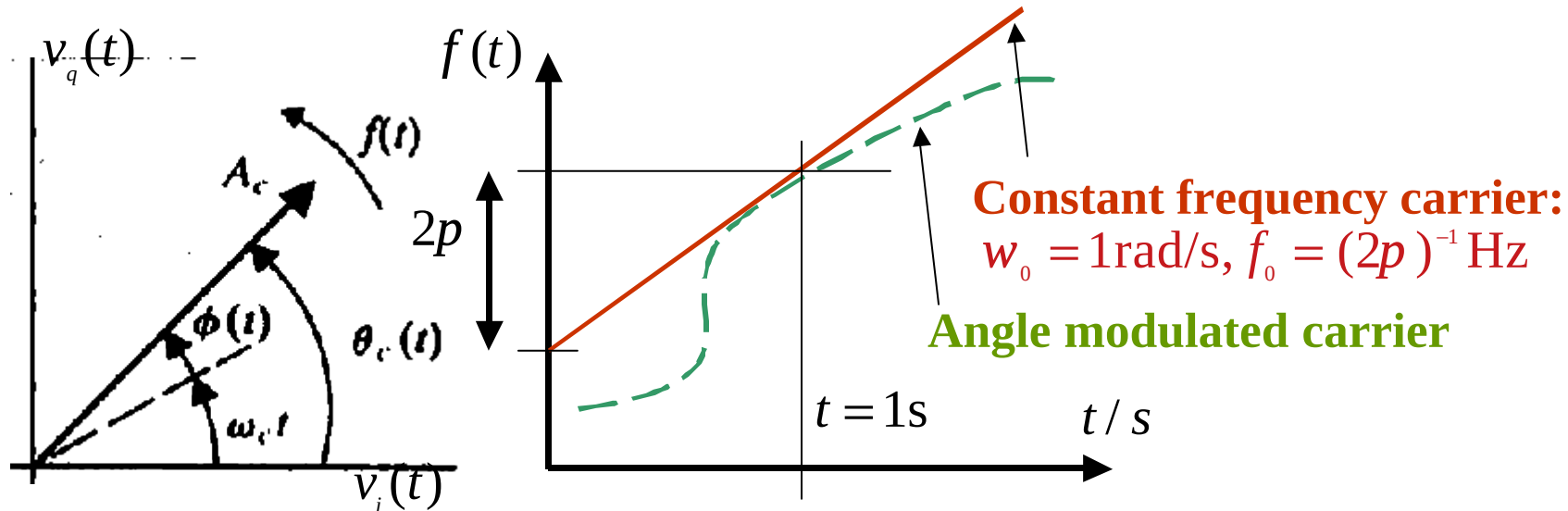
Angular phasor has the

instantaneous frequency (**phasor rate**) $q_c(t)$

$$w = 2p f(t)$$



Instantaneous frequency



- Angular frequency w (rate) is the derivative of the phase (the same way as the velocity $v(t)$ is the derivative of distance $s(t)$)
- For continuously changing frequency instantaneous frequency is defined by differential changes:

$$w(t) = \frac{df(t)}{dt} \quad f(t) = \int_{-\infty}^t w(a) da$$

Compare to linear motion:

$$v(t) = \frac{ds(t)}{dt} \left(\approx \frac{s_2(t) - s_1(t)}{t_2 - t_1} \right)$$

Frequency modulation (FM)

- v In frequency modulation carrier instantaneous frequency is linearly proportional to modulation frequency:

$$\begin{aligned}
 w &= 2\pi f(t) = dq_c(t)/dt \\
 &= 2\pi [f_c + f_\Delta x(t)]
 \end{aligned}$$

- v Hence the FM waveform can be written as

$$x_c(t) = A_c \cos(\underbrace{w_c t + 2\pi f_\Delta \int_{t_0}^t x(l) dl}_{q_c(t)}), t \geq t_0$$

$f(t) = \int_{-\infty}^t w(a) da$

←
 integrate

- v Note that for FM

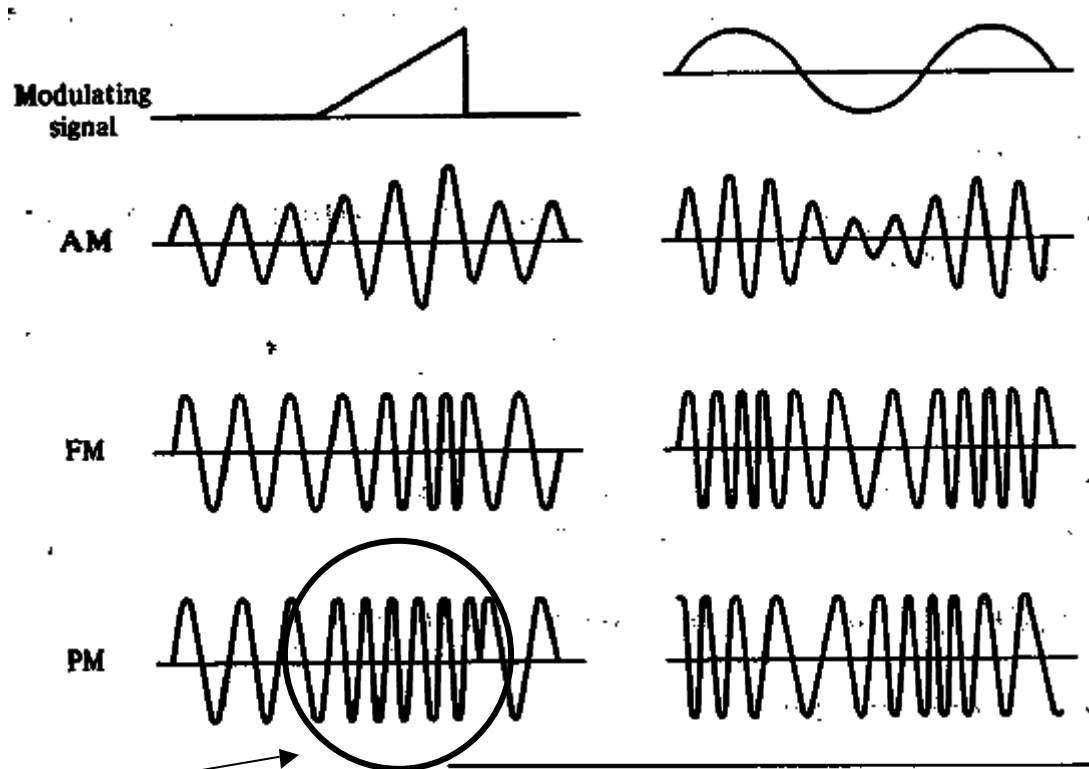
$$f(t) = f_c + f_\Delta x(t)$$

and for PM

$$f(t) = f_c + f_\Delta x(t)$$

	Instantaneous phase $\phi(t)$	Instantaneous frequency $f(t)$
PM	$\phi_\Delta x(t)$	$f_c + \frac{1}{2\pi} \phi_\Delta \dot{x}(t)$
FM	$2\pi f_\Delta \int x(\lambda) d\lambda$	$f_c + f_\Delta x(t)$

AM, FM and PM waveforms



Constant frequency follows
constant modulation waveform
derivative

	Instantaneous phase $\phi(t)$	Instantaneous frequency $f(t)$
PM	$\phi_{\Delta} x(t)$	$f_c + \frac{1}{2\pi} \phi_{\Delta} \dot{x}(t)$
FM	$2\pi f_{\Delta} \int_{-\infty}^t x(\lambda) d\lambda$	$f_c + f_{\Delta} x(t)$

$$x_{PM}(t) = A_C \cos(\omega_c t + f_{\Delta} x(t)) \quad \text{PM}$$

$$x_{FM}(t) = A_C \cos(\omega_c t + 2\pi f_{\Delta} \int_{-\infty}^t x(\lambda) d\lambda) \quad \text{FM}$$

FM Bandwidth

Normally calculated using Carlsons rule

$$B = 2 (D + 1)W$$

where W is the maximum modulation frequency and
 D is the deviation ratio $D = f_{\Delta} / W$

f_{Δ} is the peak frequency deviation

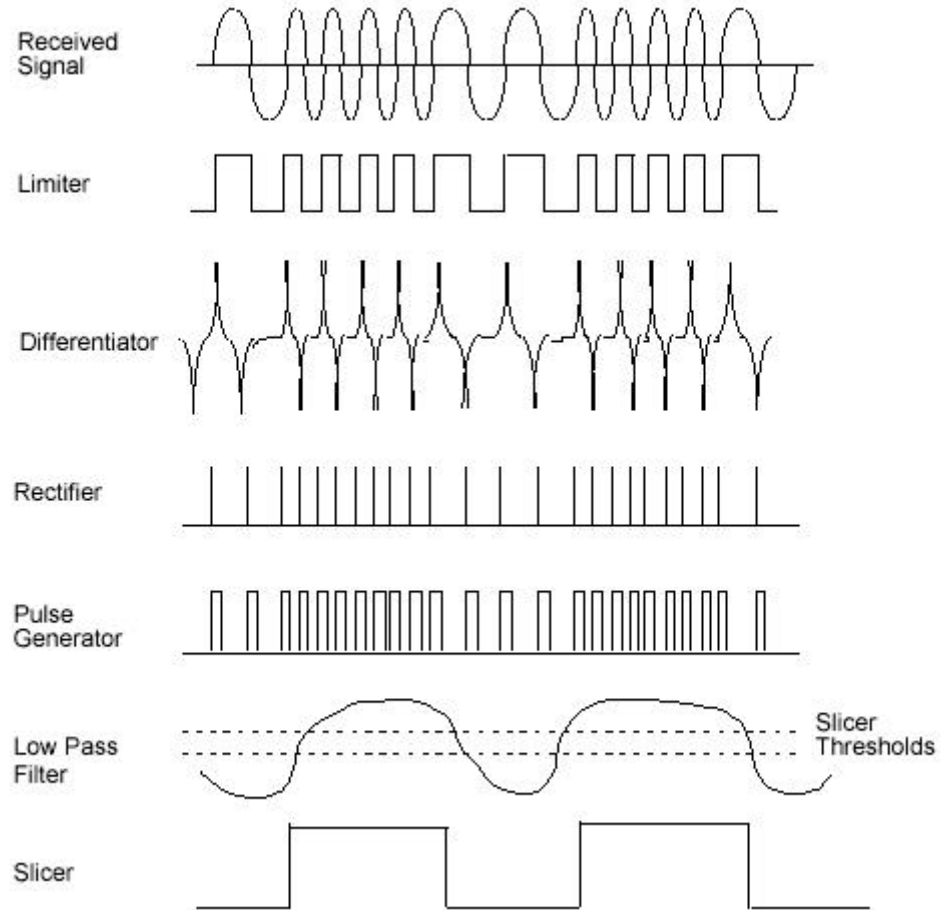
FM Radio: $f_{\Delta} = 75$ kHz, $W = 15$ kHz

$$B = 2 * (75 / 15 + 1) * 15 \text{ kHz} = 180 \text{ kHz}$$

FM demodulation - Example

Zero-crossing
based demodulation

Other:
PLL



Comparison of carrier wave modulation systems

Type	$b = B_T/W$	$(S/N)_D/\gamma$	γ_{th}	DC	Complexity	Comments	Typical applications
Baseband	1	1	...	No†	Minor	No modulation	Short-haul links
AM	2	$\frac{\mu^2 S_x}{1 + \mu^2 S_x}$	20	No	Minor	Envelope detection $\mu \leq 1$	Broadcast radio
DSB	2	1	...	Yes	Major	Synchronous detection	Analog data, multiplexing
SSB	1	1	...	No	Moderate	Synchronous detection	Point-to-point voice multiplexing
QSSB	1+	1	...	Yes	Major	Synchronous detection	Digital data
QSSB + C	1+	$\frac{\mu^2 S_x}{1 + \mu^2 S_x}$	20	Yes‡	Moderate	Envelope detection $\mu < 1$	Television video
PM§	$2M(\phi_\Delta)$	$\phi_\Delta^2 S_x$	$10b$	Yes	Moderate	Phase detection $\phi_\Delta \leq \pi$	Digital data
FM§¶	$2M(D)$	$3D^2 S_x$	$10b$	Yes	Moderate	Frequency detection	Broadcast radio, microwave relay, satellite systems

Unless direct-coupled.

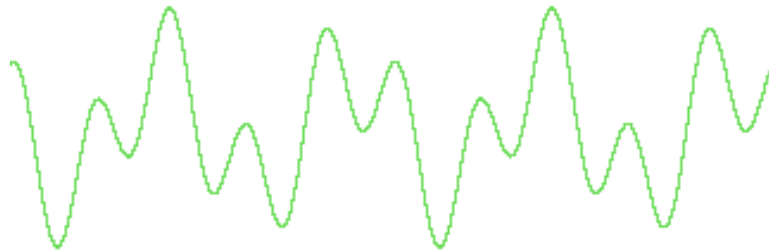
With electronic DC restoration.

$b \geq 2$.

Digital modulation

On-off keying (Binary Amplitude Key Shifting) bandwidth?

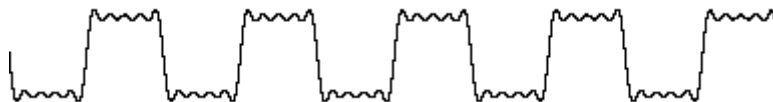
2 analog signals



digital signal (010101...)



digital signal approximated
using 5 sinus waveforms



$$\left[\sin f_m x + \frac{\sin 3\pi f_m x}{3} + \frac{\sin 5\pi f_m x}{5} + \frac{\sin 7\pi f_m x}{7} + \frac{\sin 9\pi f_m x}{9} \right] \text{ e.g. keeping 5 components } \rightarrow B = 18 f_m$$

Shannon's theorem

The capacity C of a channel is

$$C = B \log_2 \left(1 + \frac{S}{N} \right).$$

where B is the bandwidth and S/N is the signal to noise ratio (given in watts/watts)

PAL analog 6MHz studio eq. $S/N = 65$ dB $\rightarrow$ 129 Mbit/s

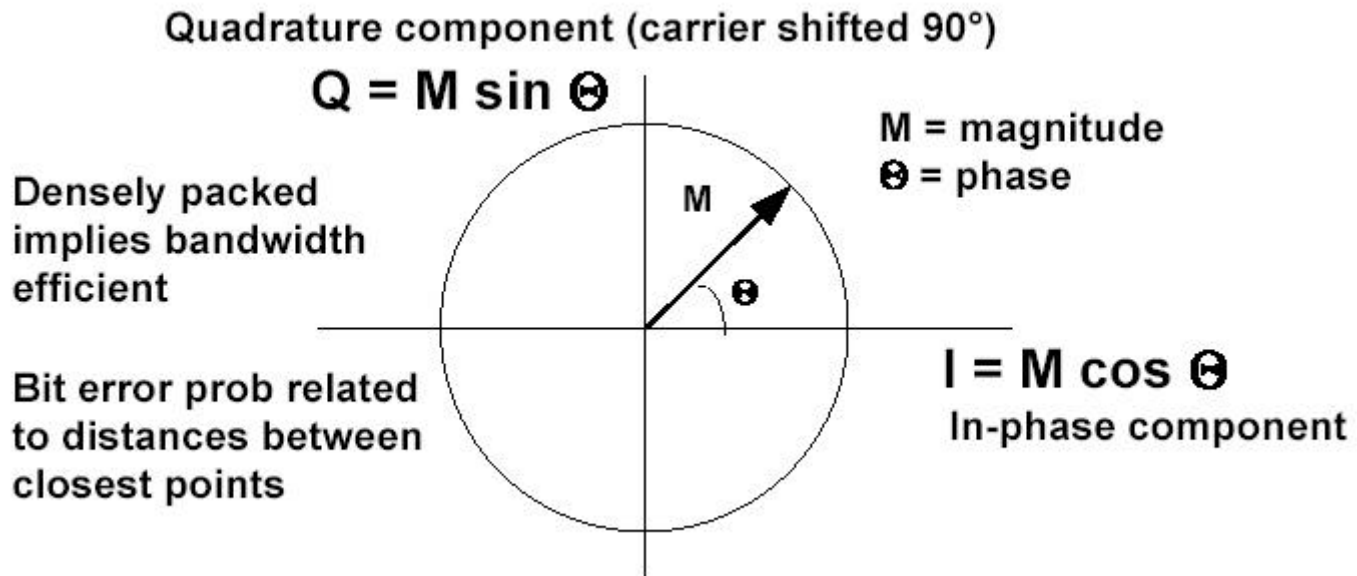
PAL analog 6MHz broadcast $S/N = 21$ dB $\rightarrow$ 42 Mbit/s

Practical for digital applications 15 dB $\rightarrow$ 30 Mbit/s

Normal digital applications: 6 bps / Hz

Digital modulation

- Modify carriers amplitude and/or phase (and frequency)
- Constellation: Vector notation / polar coordinates

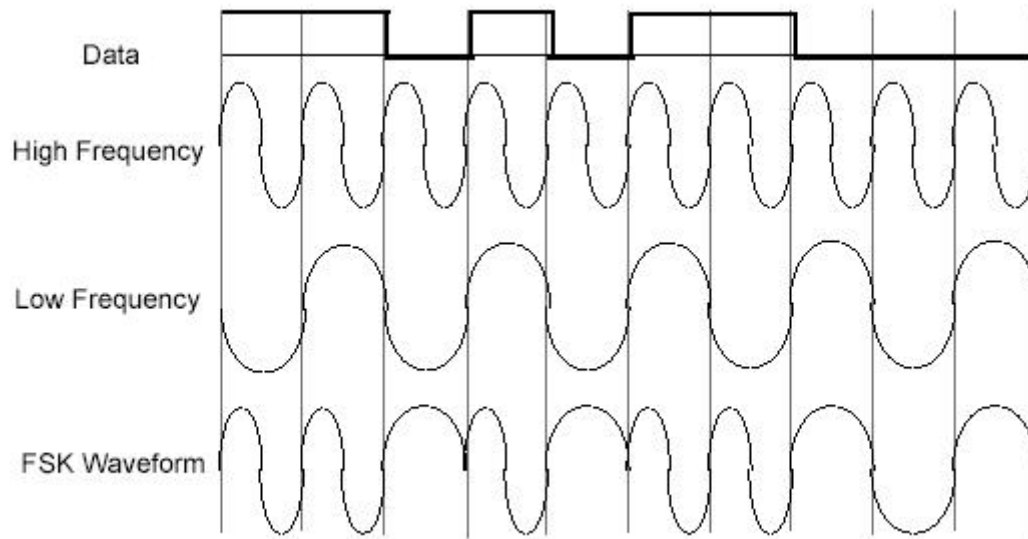
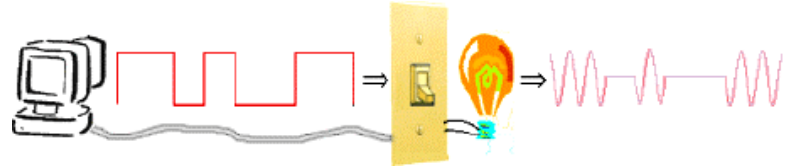


Modulation scheme - considerations

- High spectral efficiency
- High power efficiency
- Robust to multipath effects
- Low cost and ease of implementation
- Low carrier-to-cochannel interference ratio
- Low out of band radiation
- Constant or near constant envelope
 - Constant: Only phase is modulated
 - Non-constant: phase and amplitude is modulated

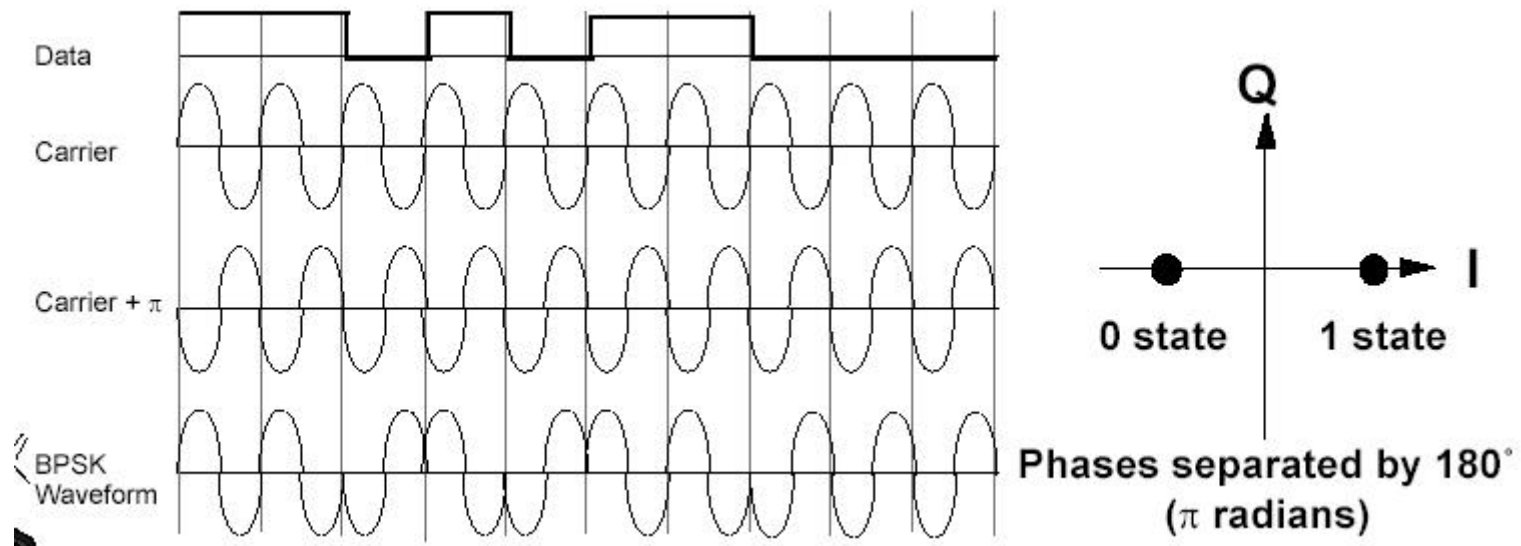
Binary modulations

- Amplitude shift keying (ASK)
Transmission on-off
- Frequency shift keying (FSK)



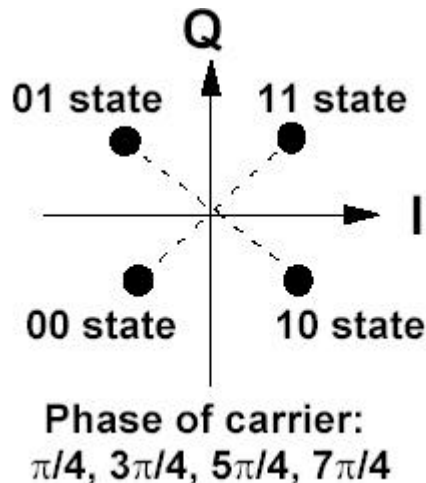
Binary modulations

- Binary phase shift keying (BPSK)
 - Simple to implement, inefficient use of bandwidth
 - Very robust, used in satellite communications



Phase key shifting

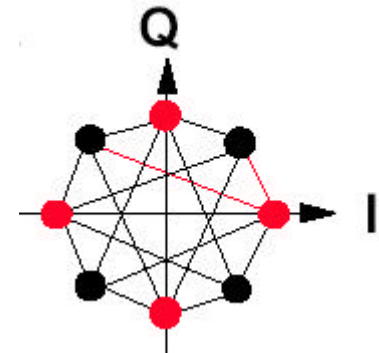
- Quadrature Phase Shift Keying (QPSK)
 - Multilevel modulation technique: 2 bits per symbol
 - More spectral efficiency, more complex receiver



Output waveform is
sum of modulated $\pm$
Cosine and $\pm$ Sine wave

$\pi / 4$ – Shifted QPSK

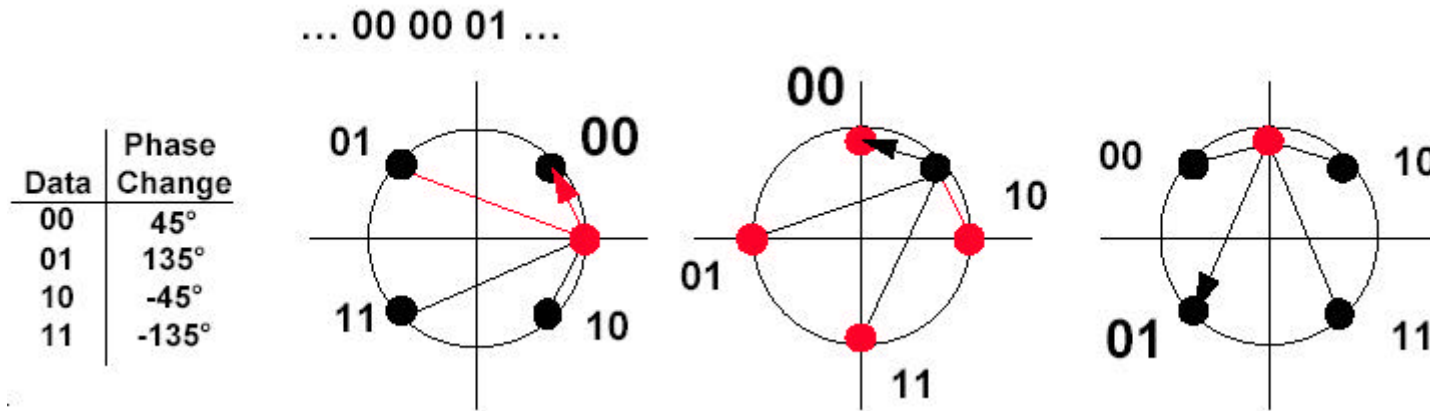
- Variation of QPSK
 - Restricted carrier phase transitions to $\pm \pi/4$ and $\pm 3\pi/4$
 - Signalling elements selected in turn from two QPSK constellations each shifted by $\pi/4$
- Popular in Second Generation Systems
 - North American Digital Cellular (1.62 bps / Hz)
 - Japanese Digital Cellular System (1.68 bps / Hz)



$\pi / 4$ – Shifted QPSK

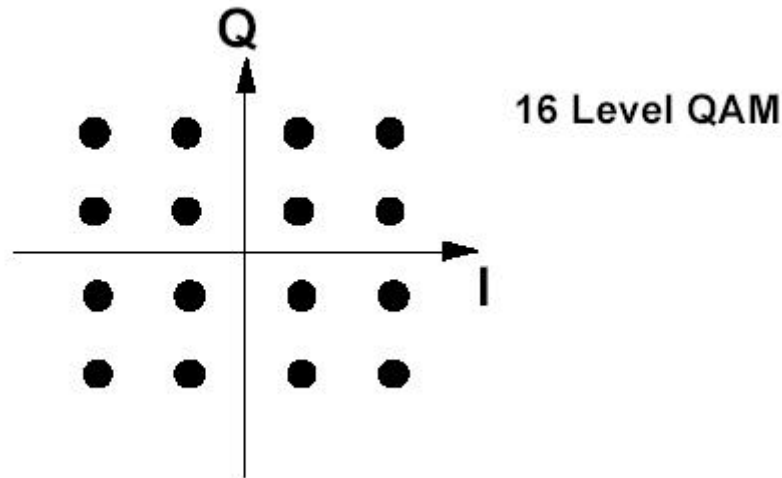
- Advantages

- Two bits per symbol
- Phase transitions avoid center of diagram, remove some design constraints on receiver
- Always a phase change between symbols, leading to self-clocking



Quadrature Amplitude Modulation

- Quadrature Amplitude Modulation (QAM)
 - Amplitude modulation on both quadrature carriers
 - 2^n discrete levels, if $n=2 \rightarrow$ same as QPSK
- Extensively used in microwave links
- DVB-T uses QAM



Quadrature Amplitude Modulation

4 bits / symbol

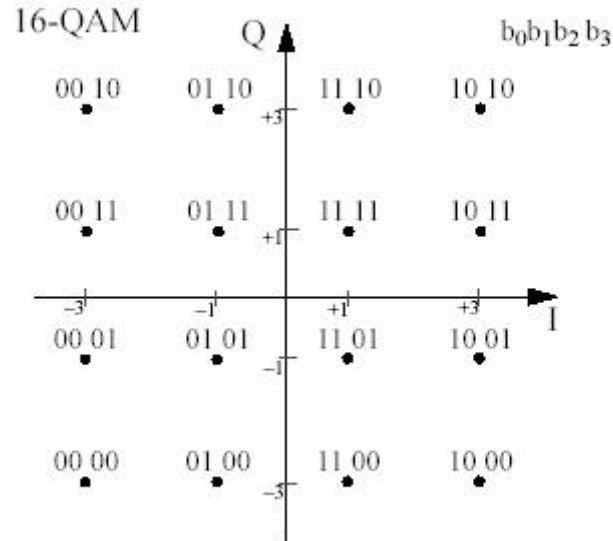


Table 84—16-QAM encoding table

Input bits ($b_0 b_1$)	I-out	Input bits ($b_2 b_3$)	Q-out
00	-3	00	-3
01	-1	01	-1
11	1	11	1
10	3	10	3

Quadrature Amplitude Modulation

6 bits / symbol

Table 85—64-QAM encoding table

Input bits ($b_0 b_1 b_2$)	I-out	Input bits ($b_3 b_4 b_5$)	Q-out
000	-7	000	-7
001	-5	001	-5
011	-3	011	-3
010	-1	010	-1
110	1	110	1
111	3	111	3
101	5	101	5
100	7	100	7

